Available online at www.sciencedirect.com
ScienceDirect
ELSEVIER
Journal of Sound and Vibration 311 (2008) 912–923
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Vibration analysis for a coupled beam-sdof system by using the recurrence equation method

H.B. Tang*, C.J. Wu, X.Q. Huang

Institute of Vibration and Noise Control, School of Mechanical Engineering, Xi'an Jiaotong University, Xi'an 710049, PR China

Received 20 August 2007; received in revised form 20 August 2007; accepted 25 September 2007
Available online 5 November 2007

Abstract

This paper presents the vibration analysis of a coupled system composed of a beam and a single-degree-of-freedom (sdof) system. The harmonic responses of this kind of system are formulated by means of the recurrence equation method. In addition, the natural characteristics of the system are also analyzed. Numerical examples of the undamped system are given and compared with the published results, and an excellent correlation shows that the study work in this paper is correct.

1. Introduction

There are many engineering problems could be modelled as the coupled beam-sdof system, such as a motor or engine elastically mounted on a structural element. The other applications could be seen in mechanical, naval and ocean engineering fields. Understanding the problem clearly is very important to the further researches such as the prediction of sound radiation from the aforementioned object. This kind of problem and the similar subjects have been studied by many researchers in the past and several effective methods have been developed. Wu and Lin [1] performed the vibration analysis of a uniform cantilever beam with point masses by an analytical-and-numerical-combined method (ANCM). Wu and Chen [2] studied the free and forced vibration of a uniform cantilever beam carrying a number of spring–damper–mass systems with arbitrary magnitudes and locations employing ANCM. The frequency equations of a Bernoulli–Euler beam to which several spring–mass systems are attached in span were investigated by Gürgöze [3]. Inceoğlu and Gürgöze [4] studied the longitudinal vibrations of rods coupled by several spring-mass systems employing the Green function method. Inceoğlu and Gürgöze [5] performed the vibration analysis of beams coupled by several double spring–mass systems. Bambill and Rossit [6] researched the forced vibrations of a beam elastically restrained against rotation and carrying a spring–mass system and gave many numerical examples with different boundary conditions. The approach presented in Ref. [6] was based on the method which divided the beam into segments from the point attached to the spring–mass system. Under the appropriate boundary conditions, the dynamic characteristics of whole system could be obtained by solving the every

*Corresponding author. Tel.: +86 29 82668573; fax: +86 29 82660487.
E-mail address: hbtang@mail.xjtu.edu.cn (H.B. Tang).

0022-460X/$ - see front matter
doi:10.1016/j.jsv.2007.09.034

segment. However, this kind of method is complicated and inconvenient for the prediction of sound radiation from this kind of system.

In this paper, a new approach is presented to analyze a coupled system which is composed of a beam and an sdof system based on the recurrence equation method. In Sections 2 and 3, the theoretical model of this kind of problem is developed. The vibration response of the coupled system is achieved by using the recurrence equation method in Section 4. The natural characteristics of the whole system are studied in Section 5. The numerical examples are given and compared with the published results [6] in Section 6. To show the utility of the present approach, the lowest four natural frequencies and mode shapes of the coupled system are also addressed. The recurrence equation method in this paper is firstly introduced into the area of the vibration analysis.

2. Statement of the problem

The problem under consideration is schematically depicted in Fig. 1: a simply supported uniform beam carrying a spring–damper–mass system at $x = a$. A system of Cartesian coordinates (x, y) is used to define the position of points of the system. A harmonic excitation $F\mathrm{e}^{-\mathrm{j}\omega t}$ is applied to the mass block, where m, c and k are the mass, damping coefficient and stiffness of the sdof system, respectively. u_1 represents the displacement of the mass block and u_2 represents the displacement of the attaching point (i.e. the displacement of the beam at $x = a$). $F_a\mathrm{e}^{-\mathrm{j}\omega t}$ denotes the transmission excitation owing to the sdof system. L is the length of the beam. It is noted that the vibration response of this coupled system could be solved by combining the equations of motion of the beam and sdof system together and by using the recurrence equation method.

3. Model development

According to the classical theory of Bernoulli–Euler beam, the transverse vibration equation of motion of the beam is well known as

$$EJ\frac{\partial^4 y(x,t)}{\partial x^4} + \rho A\frac{\partial^2 y(x,t)}{\partial t^2} = F_a\mathrm{e}^{-\mathrm{j}\omega t}\delta(x-a), \tag{1}$$

where E, A, J and ρ are Young's modulus, the cross–sectional area of the beam, the moment of inertial and the density of the beam material, respectively. δ is the Dirac–delta distribution function. $y(x, t)$ denotes the transverse deflection of the beam.

Assuming $y(x,t) = Y(x)\mathrm{e}^{-\mathrm{j}\omega t}$, for the simply supported beam, the steady–state response of the beam under harmonic excitation can be written as [7]

$$Y(x) = \frac{-2F_a}{\rho AL}\sum_{n=1}^{\infty}\frac{\sin(n\pi a/L)\sin(n\pi x/L)}{\omega^2-\omega_n^2}, \tag{2}$$

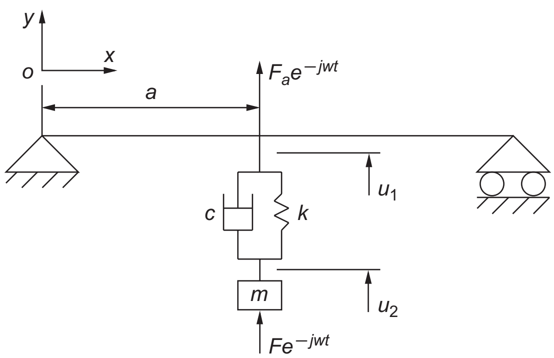

Fig. 1. Coupled beam-sdof system and coordinate system.

where ω_n is the nth natural frequency of the bare beam (i.e. the beam without the sdof system), $\omega_n = n^2\pi^2/L^2\sqrt{EJ/\rho A}$.

Subsequently, we will study the response of the whole system. One can get the differential equation of motion of the spring–damper–mass system as follows:

$$m\ddot{u}_1 + c(\dot{u}_1 - \dot{u}_2) + k(u_1 - u_2) = F\mathrm{e}^{-\mathrm{j}\omega t}. \tag{3}$$

Let $u = u_1 - u_2$, one can get

$$m\ddot{u} + c\dot{u} + ku = F\mathrm{e}^{-\mathrm{j}\omega t} - m\ddot{u}_2. \tag{4}$$

Because $u_2 = Y(a)\mathrm{e}^{-\mathrm{j}\omega t}$, solving Eq. (4), one can obtain

$$u = \frac{F + m\omega^2 Y(a)}{k - m\omega^2 - \mathrm{j}\omega c}\mathrm{e}^{-\mathrm{j}\omega t}. \tag{5}$$

According to the force equilibrium of $F_a\mathrm{e}^{-\mathrm{j}\omega t} = ku + c\dot{u}$, the harmonic excitation can be expressed as

$$F_a = \frac{F + m\omega^2 Y(a)}{k - m\omega^2 - \mathrm{j}\omega c}(k - \mathrm{j}\omega c). \tag{6}$$

It can be seen that Eqs. (2) and (6) are coupled. Consequently, it is difficult to solve Eqs. (2) and (6) simultaneously in general ways. Because of the essential feedback effect, this kind of problem can be solved by means of the recurrence equation method.

4. Solving the response

In this section, we will solve the response of the coupled system by using the recurrence equation method. From Eq. (2), the transverse deflection of the beam at $x = a$ is given by

$$Y(a) = \frac{-2F_a}{\rho AL}\sum_{n=1}^{\infty}\frac{\sin^2(n\pi a/L)}{\omega^2 - \omega_n^2}. \tag{7}$$

Inserting Eq. (6) into Eq. (7), one can obtain the recurrence equation as

$$Y_{s+1}(a) = \left(\frac{m\omega^2 Y_s(a) + F}{k - m\omega^2 - \mathrm{j}\omega c}(k - \mathrm{j}\omega c)\right)\left(\frac{-2}{\rho AL}\sum_{n=1}^{\infty}\frac{\sin^2(n\pi a/L)}{\omega^2 - \omega_n^2}\right), \tag{8}$$

where s is the times of recurrence.

Eq. (8) can be rewritten in the form of

$$Y_{s+1}(a) = D(GY_s(a) + FH), \tag{9}$$

where

$$D = \frac{1}{c^2\omega^2 + (k - m\omega^2)^2}\frac{-2}{\rho AL}\sum_{n=1}^{\infty}\frac{\sin^2(n\pi a/L)}{\omega^2 - \omega_n^2}, \tag{10}$$

$$G = k^2 m\omega^2 + c^2 m\omega^4 - km^2\omega^4 + \mathrm{j}cm^2\omega^5, \tag{11}$$

$$H = k^2 + c^2\omega^2 - km\omega^2 + \mathrm{j}cm\omega^3. \tag{12}$$

Solving Eq. (9) gives

$$Y_s(a) = -\frac{FHD(1 - (GD)^s)}{GD - 1}, \tag{13}$$

where $Y_0(a) = 0$, which is the initial condition of the system at $t = 0$.

The main interest is focused on the steady–state response, so the final transverse deflection of the beam at $x = a$ can be expressed as

$$Y(a) = \lim_{s\to\infty} -\frac{FHD(1-(GD)^s)}{GD-1}. \tag{14}$$

When $|GD|>1$, $Y(a)$ will diverge obviously. For the case $GD = 1$, the physical meanings will be studied in the following section. When $|GD|<1$, one can obtain the converged solution as

$$Y(a) = -\frac{FHD}{GD-1}. \tag{15}$$

Combining Eq. (15) with Eqs. (2) and (6) gives

$$Y(x) = \frac{FH/(1-GD)}{c^2\omega^2+(k-m\omega^2)^2}\frac{-2}{\rho AL}\sum_{n=1}^{\infty}\frac{\sin(n\pi a/L)\sin(n\pi x/L)}{\omega^2-\omega_n^2}. \tag{16}$$

Eq. (16) is the analytical solution of transverse amplitudes for a uniform beam carrying a spring–damper–mass system in the steady–state situation.

For the undamped system, the transverse deflection of the attaching point is given by

$$Y(a) = \frac{F}{m\omega^2}\frac{-C}{C-1}, \tag{17}$$

where

$$C = \frac{1}{(1/k)-(1/m\omega^2)}\frac{2}{\rho AL}\sum_{n=1}^{\infty}\frac{\sin^2(n\pi a/L)}{\omega^2-\omega_n^2}. \tag{18}$$

Substituting Eq. (17) into Eqs. (2) and (6) gives

$$Y(x) = \frac{(F/m\omega^2)(1/C-1)}{(1/k)-(1/m\omega^2)}\frac{-2}{\rho AL}\sum_{n=1}^{\infty}\frac{\sin(n\pi a/L)\sin(n\pi x/L)}{\omega^2-\omega_n^2}. \tag{19}$$

Eq. (19) is the analytical solution of the undamped system. Compared with undamped system, the amplitudes of the damped system (see Eq. (16)) are complex values.

5. Natural characteristics of the whole system

Next, the natural characteristics of the whole system are analyzed. From Eq. (9), let $F = 0$ one can obtain the recurrence equation as

$$Y_{s+1}(a) = GDY_s(a). \tag{20}$$

Solving Eq. (20) gives

$$Y_s(a) = (GD)^s\varepsilon, \tag{21}$$

where ε is the initial deflection of the attaching point and $\varepsilon \neq 0$.

For the purpose of keeping the state of vibration, it must be satisfied of

$$GD = 1. \tag{22}$$

Solving Eq. (22), the lth natural frequency $\bar{\omega}_l$ of the whole system can be achieved. Substituting Eq. (22) into Eq. (21), one obtains

$$Y(a) = \varepsilon. \tag{23}$$

Combining Eq. (23) with Eqs. (2) and (6), let $F=0$, $\varepsilon=1$ and $\omega=\bar{\omega}_l$, the mode shapes can be expressed as

$$Y_l(x)=\frac{k^2m\bar{\omega}_l^2+c^2m\bar{\omega}_l^4-km^2\bar{\omega}_l^4+\mathrm{j}cm^2\bar{\omega}_l^5}{c^2\bar{\omega}_l^2+(k-m\bar{\omega}_l^2)^2}\frac{-2}{\rho AL}\sum_{n=1}^{\infty}\frac{\sin(n\pi a/L)\sin(n\pi x/L)}{\bar{\omega}_l^2-\omega_n^2},\ l=1,2,\cdots. \tag{24}$$

Following the same way, solving the equation of (see Eq. (18))

$$C=1, \tag{25}$$

the lth natural frequency $\bar{\omega}_l$ of the undamped system can be reached. And the corresponding mode shapes of the undamped system can be given by

$$Y_l(x)=\frac{1}{(1/k)-(1/m\bar{\omega}_l^2)}\frac{2}{\rho AL}\sum_{n=1}^{\infty}\frac{\sin(n\pi a/L)\sin(n\pi x/L)}{\bar{\omega}_l^2-\omega_n^2},\quad l=1,2,\cdots. \tag{26}$$

Table 1
The dimensionless vibration amplitude values of $EJY(x)/FL^3$ when $M=1$ and $K=1$

a/L ($\bar{\omega}_1/\omega_1$)	x	$EJY(x)/FL^3$ ω/ω_1						
		0.10	0.30	0.50	0.70	0.90	1.10	1.30
0.1	0	0	0	0	0	0	0	0
(0.101184)	0.1	0.1169530	−0.0003717	−0.0001434	−0.0000980	−0.0001407	0.0000739	0.0000125
	0.2	0.2022860	−0.0006468	−0.0002527	−0.0001762	−0.0002616	0.0001447	0.0000268
	0.3	0.2530570	−0.0008143	−0.0003224	−0.0002298	−0.0003523	0.0002044	0.0000407
	0.4	0.2735040	−0.0008854	−0.0003550	−0.0002579	−0.0004068	0.0002454	0.0000516
	0.5	0.2678940	−0.0008719	−0.0003534	−0.0002612	−0.0004216	0.0002623	0.0000574
	0.6	0.2405170	−0.0007864	−0.0003216	−0.0002410	−0.0003965	0.0002527	0.0000570
	0.7	0.1956910	−0.0006421	−0.0002646	−0.0002004	−0.0003344	0.0002169	0.0000499
	0.8	0.1377540	−0.0004532	−0.0001877	−0.0001433	−0.0002416	0.0001586	0.0000370
	0.9	0.0710666	−0.0002348	−0.0000973	−0.0000746	−0.0001266	0.0000837	0.0000197
	1	0	0	0	0	0	0	0
0.3	0	0	0	0	0	0	0	0
(0.100578)	0.1	0.5137730	−0.0008024	−0.0003172	−0.0002245	−0.0003321	0.0002163	0.0000414
	0.2	0.9659820	−0.0015098	−0.0005978	−0.0004242	−0.0006299	0.0004128	0.0000797
	0.3	1.2951100	−0.0020279	−0.0008060	−0.0005754	−0.0008620	0.0005721	0.0001125
	0.4	1.4543000	−0.0022848	−0.0009142	−0.0006597	−0.0010036	0.0006802	0.0001376
	0.5	1.4549700	−0.0022944	−0.0009251	−0.0006755	−0.0010446	0.0007235	0.0001506
	0.6	1.3232800	−0.0020940	−0.0008503	−0.0006276	−0.0009847	0.0006949	0.0001480
	0.7	1.0854900	−0.0017228	−0.0007037	−0.0005239	−0.0008318	0.0005958	0.0001292
	0.8	0.7679920	−0.0012215	−0.0005011	−0.0003755	−0.0006013	0.0004352	0.0000956
	0.9	0.3973040	−0.0006328	−0.0002603	−0.0001958	−0.0003152	0.0002296	0.0000508
	1	0	0	0	0	0	0	0
0.5	0	0	0	0	0	0	0	0
(0.100270)	0.1	1.1512400	−0.0008527	−0.0003447	−0.0002520	−0.0003842	0.0002884	0.0000591
	0.2	2.2089500	−0.0016350	−0.0006599	−0.0004814	−0.0007320	0.0005476	0.0001117
	0.3	3.0796800	−0.0022772	−0.0009172	−0.0006670	−0.0010100	0.0007517	0.0001523
	0.4	3.6702300	−0.0027109	−0.0010895	−0.0007896	−0.0011904	0.0008811	0.0001773
	0.5	3.8876700	−0.0028698	−0.0011520	−0.0008335	−0.0012535	0.0009249	0.0001855
	0.6	3.6702300	−0.0027109	−0.0010895	−0.0007896	−0.0011904	0.0008811	0.0001773
	0.7	3.0796800	−0.0022771	−0.0009172	−0.0006670	−0.0010100	0.0007517	0.0001523
	0.8	2.2089500	−0.0016350	−0.0006599	−0.0004814	−0.0007320	0.0005476	0.0001117
	0.9	1.1512400	−0.0008527	−0.0003447	−0.0002520	−0.0003842	0.0002884	0.0000591
	1	0	0	0	0	0	0	0

Table 2
The dimensionless vibration amplitude values of $EJY(x)/FL^3$ when $M = 0.1$ and $K = 2$

a/L ($\bar{\omega}_1/\omega_1$)	x	$EJY(x)/FL^3$ ω/ω_1						
		0.10	0.30	0.50	0.70	0.90	1.10	1.30
0.1	0	0	0	0	0	0	0	0
(0.451675)	0.1	0.0028602	0.0051807	−0.0148856	−0.0032652	−0.0036551	0.0017857	0.0002842
	0.2	0.0049471	0.0090138	−0.0262259	−0.0058742	−0.0067950	0.0034971	0.0006083
	0.3	0.0061887	0.0113484	−0.0334625	−0.0076593	−0.0091524	0.0049418	0.0009244
	0.4	0.0066888	0.0123393	−0.0368378	−0.0085985	−0.0105678	0.0059332	0.0011713
	0.5	0.0065516	0.0121512	−0.0366738	−0.0087054	−0.0109524	0.0063421	0.0013029
	0.6	0.0058820	0.0109592	−0.0333808	−0.0080344	−0.0102995	0.0061082	0.0012922
	0.7	0.0047858	0.0089492	−0.0274572	−0.0066806	−0.0086876	0.0052444	0.0011331
	0.8	0.0033689	0.0063165	−0.0194816	−0.0047769	−0.0062750	0.0038347	0.0008404
	0.9	0.0017380	0.0032639	−0.0100987	−0.0024878	−0.0032876	0.0020235	0.0004471
	1	0	0	0	0	0	0	0
0.3	0	0	0	0	0	0	0	0
(0.445164)	0.1	0.0061964	0.0115871	−0.0285810	−0.0071044	−0.0078993	0.0056998	0.0009596
	0.2	0.0116503	0.0218023	−0.0538638	−0.0134241	−0.0149856	0.0108769	0.0018473
	0.3	0.0156198	0.0292835	−0.0726188	−0.0182077	−0.0205069	0.0150744	0.0026071
	0.4	0.0175396	0.0329920	−0.0823750	−0.0208764	−0.0238750	0.0179220	0.0031887
	0.5	0.0175478	0.0331313	−0.0833564	−0.0213757	−0.0248504	0.0190637	0.0034883
	0.6	0.0159595	0.0302379	−0.0766146	−0.0198587	−0.0234262	0.0183114	0.0034296
	0.7	0.0130916	0.0248768	−0.0634009	−0.0165788	−0.0197885	0.0156978	0.0029928
	0.8	0.0092624	0.0176391	−0.0451507	−0.0118832	−0.0143053	0.0114680	0.0022137
	0.9	0.0047917	0.0091375	−0.0234517	−0.0061966	−0.0074982	0.0060489	0.0011762
	1	0	0	0	0	0	0	0
0.5	0	0	0	0	0	0	0	0
(0.441859)	0.1	0.0065639	0.0125439	−0.0290542	−0.0077597	−0.0087401	0.0081170	0.0013900
	0.2	0.0125944	0.0240515	−0.0556295	−0.0148260	−0.0166523	0.0154111	0.0026282
	0.3	0.0175590	0.0334975	−0.0773168	−0.0205419	−0.0229766	0.0211538	0.0035851
	0.4	0.0209260	0.0398773	−0.0918404	−0.0243201	−0.0270823	0.0247947	0.0041737
	0.5	0.0221658	0.0422151	−0.0971102	−0.0256699	−0.0285170	0.0260290	0.0043653
	0.6	0.0209260	0.0398773	−0.0918404	−0.0243201	−0.0270823	0.0247947	0.0041737
	0.7	0.0175590	0.0334975	−0.0773168	−0.0205419	−0.0229766	0.0211538	0.0035851
	0.8	0.0125944	0.0240515	−0.0556295	−0.0148260	−0.0166523	0.0154111	0.0026282
	0.9	0.0065639	0.0125439	−0.0290542	−0.0077597	−0.0087401	0.0081170	0.0013900
	1	0	0	0	0	0	0	0

6. Numerical results

This section is devoted to the numerical evaluations of the formulae established in the preceding sections. At the beginning of, the numerical examples of the undamped system are given in order to compare with the results available in literature [6]. The geometrical and mechanical parameter values of the coupled system are as follows: $L = 1.0\,\text{m}$, $EJ = 6 \times 10^4\,\text{N}\,\text{m}^2$ and $\rho A = 15\,\text{kg/m}$.

The dimensionless parameters of M and K are employed in the following study where $M = m/\rho AL$ and $K = kL^3/EJ$. For the sake of brevity, the dimensionless vibration amplitude $EJY(x)/FL^3$ is introduced. In Tables 1–4, the values of $\bar{\omega}_1/\omega_1$ are indicated, where $\bar{\omega}_1$ and ω_1 are the fundamental natural frequency of the coupled beam and the bare beam respectively.

Tables 1 and 2 show the values of $EJY(x)/FL^3$ when $M = 1$, $K = 1$ and $M = 0.1$, $K = 2$ respectively. Tables 3 and 4 are cited in Ref. [6]. Compared with Tables 1–4, it is noted that the agreements between Tables 1 and 3 and Tables 2 and 4 are excellent. The comparison shows the study work in this paper is correct. Also it is noticed that the value is −0.0148856 in Table 2 when $a/L = 0.1$ and $\omega/\omega_1 = 0.5$, and its counterpart in Table 4 is 0.1488573. The authors think that this is a slip of the pen in Ref. [6].

Table 3
The dimensionless vibration amplitude values of $EJY(x)/FL^3$ when $M = 1$ and $K = 1$ (cited in Ref. [6])

a/L ($\bar{\omega}_1/\omega_1$)	x	$EJY(x)/FL^3$ ω/ω_1						
		0.10	0.30	0.50	0.70	0.90	1.10	1.30
0.1	0	0	0	0	0	0	0	0
(0.10118)	0.1	0.1169460	0.0003717	0.0001434	0.0000980	0.0001407	0.0000739	0.0000125
	0.2	0.2022750	0.0006468	0.0002527	0.0001762	0.0002616	0.0001447	0.0000268
	0.3	0.2530430	0.0008143	0.0003224	0.0002298	0.0003523	0.0002044	0.0000408
	0.4	0.2734890	0.0008854	0.0003550	0.0002580	0.0004068	0.0002454	0.0000516
	0.5	0.2678790	0.0008719	0.0003534	0.0002612	0.0004216	0.0002623	0.0000574
	0.6	0.2405040	0.0007864	0.0003216	0.0002410	0.0003965	0.0002527	0.0000570
	0.7	0.1956800	0.0006421	0.0002646	0.0002004	0.0003344	0.0002169	0.0000500
	0.8	0.1377470	0.0004532	0.0001877	0.0001433	0.0002416	0.0001586	0.0000370
	0.9	0.0710626	0.0002342	0.0000973	0.0000746	0.0001266	0.0000837	0.0000197
	1	0	0	0	0	0	0	0
0.3	0	0	0	0	0	0	0	0
(0.10058)	0.1	0.5137140	0.0008024	0.0003172	0.0002245	0.0003321	0.0002163	0.0000414
	0.2	0.9658730	0.0015098	0.0005978	0.0004242	0.0006299	0.0004128	0.0000797
	0.3	1.2949700	0.0020279	0.0008060	0.0005754	0.0008620	0.0005721	0.0001125
	0.4	1.4541300	0.0022848	0.0009143	0.0006597	0.0010036	0.0006802	0.0001376
	0.5	1.4548100	0.0022944	0.0009251	0.0006755	0.0010446	0.0007235	0.0001506
	0.6	1.3231300	0.0020940	0.0008503	0.0006276	0.0009847	0.0006950	0.0001480
	0.7	1.0853600	0.0017228	0.0007037	0.0005239	0.0008318	0.0005958	0.0001292
	0.8	1.0853600	0.0012215	0.0005011	0.0003755	0.0006013	0.0004352	0.0000956
	0.9	0.3972590	0.0006328	0.0002603	0.0001958	0.0003152	0.0002296	0.0000508
	1	0	0	0	0	0	0	0
0.5	0	0	0	0	0	0	0	0
(0.10027)	0.1	1.1509700	0.0008527	0.0003447	0.0002520	0.0003842	0.0002884	0.0000591
	0.2	2.2084200	0.0016350	0.0006599	0.0004814	0.0007320	0.0005476	0.0001117
	0.3	3.0789400	0.0022772	0.0009172	0.0006670	0.0010100	0.0007517	0.0001523
	0.4	3.6693500	0.0027109	0.0010895	0.0007896	0.0011904	0.0008811	0.0001773
	0.5	3.8867400	0.0028698	0.0011520	0.0008335	0.0012535	0.0009249	0.0001855
	0.6	3.6693500	0.0027109	0.0010895	0.0007896	0.0011904	0.0008811	0.0001773
	0.7	3.0789400	0.0022772	0.0009172	0.0006670	0.0010100	0.0007517	0.0001523
	0.8	2.2084200	0.0016350	0.0006599	0.0004814	0.0007320	0.0005476	0.0001117
	0.9	1.1509700	0.0008527	0.0003447	0.0002520	0.0003842	0.0002884	0.0000591
	1	0	0	0	0	0	0	0

The dimensionless vibration amplitudes of the whole system are plotted for different M, K, ω and ξ (ξ is the damping ratio, $\xi = c/(2\sqrt{mk})$) at $x/L = 0.5$ depending on Eq. (16), which are shown in Figs. 2–5, respectively. In Fig. 2, the amplitude curves are shown when M is varying from 0 to 1. Note that there are peaks when M is around 0.2. Fig. 3 shows the amplitude versus K when M, ξ, ω and a/L are fixed. Also there are peaks when K is close to 0.5. In the two cases, it can be supposed that the position of the peak is little dependent of a/L, which is a meaningful conclusion for the vibration analysis of this kind of model. The amplitude–frequency responses in the frequency range $0 - 2000\,\text{rad/s}$ are illustrated in Fig. 4, where two resonant peaks (i.e. $\omega = 175$ and $623\,\text{rad/s}$) are present clearly. In the identical situation, the corresponding natural frequencies derived from Eq. (22) are 174.63 and 623.26 rad/s when $a/L = 0.5$, which are in agreement with the resonant frequencies shown in Fig. 4. Further, it can be seen in Fig. 5 that the amplitudes and damping ratio vary reversely. From the viewpoint of vibration control, this peak values mentioned before should be avoided.

Figs. 6–9 show the lowest four mode shapes of the undamped system when $M = 0.1$ and $K = 2$. It can be seen that the first mode shapes in Fig. 6 are similar with the second ones in Fig. 7. Meanwhile,

Table 4
The dimensionless vibration amplitude values of $EJY(x)/FL^3$ when $M = 0.1$ and $K = 2$ (cited in Ref. [6])

a/L ($\bar{\omega}_1/\omega_1$)	x	$EJY(x)/FL^3$ ω/ω_1						
		0.10	0.30	0.50	0.70	0.90	1.10	1.30
0.1	0	0	0	0	0	0	0	0
(0.45167)	0.1	0.0028602	0.0051807	0.1488573	0.0032652	0.0036551	0.0017857	0.0002842
	0.2	0.0049471	0.0090138	0.0262260	0.0058742	0.0067949	0.0034971	0.0006083
	0.3	0.0061887	0.0113484	0.0334628	0.0076593	0.0091524	0.0049418	0.0009244
	0.4	0.0066888	0.0123393	0.0368381	0.0085985	0.0105677	0.0059332	0.0011713
	0.5	0.0065516	0.0121511	0.0366740	0.0087054	0.0109523	0.0063422	0.0013029
	0.6	0.0058820	0.0109592	0.0333810	0.0080344	0.0102995	0.0061082	0.0012922
	0.7	0.0047858	0.0089492	0.0274574	0.0066806	0.0086876	0.0052444	0.0011331
	0.8	0.0033689	0.0063164	0.0194817	0.0047769	0.0062750	0.0038347	0.0008404
	0.9	0.0017380	0.0032639	0.0100988	0.0024878	0.0032876	0.0020236	0.0004471
	1	0	0	0	0	0	0	0
0.3	0	0	0	0	0	0	0	0
(0.44516)	0.1	0.0061964	0.0115871	0.0285811	0.0071044	0.0078993	0.0056999	0.0009596
	0.2	0.0116503	0.0218022	0.0538642	0.0134241	0.0149855	0.0108770	0.0018473
	0.3	0.0156198	0.0292835	0.0726192	0.0182077	0.0205068	0.0150746	0.0026071
	0.4	0.0175396	0.0329920	0.0823754	0.0208765	0.0238749	0.0179222	0.0031887
	0.5	0.0175478	0.0331313	0.0833569	0.0213757	0.0248503	0.0190639	0.0034883
	0.6	0.0159595	0.0302379	0.0766150	0.0198587	0.0234261	0.0183116	0.0034296
	0.7	0.0130916	0.0248767	0.0634013	0.0165788	0.0197885	0.0156980	0.0029929
	0.8	0.0092624	0.0176390	0.0451510	0.0118832	0.0143052	0.0114682	0.0022137
	0.9	0.0047917	0.0091375	0.0234518	0.0061966	0.0074981	0.0060489	0.0011762
	1	0	0	0	0	0	0	0
0.5	0	0	0	0	0	0	0	0
(0.44186)	0.1	0.0065639	0.0125439	0.0290543	0.0077597	0.0087401	0.0081171	0.0013900
	0.2	0.0125944	0.0240515	0.0556298	0.0148260	0.0166523	0.0154113	0.0026282
	0.3	0.0175590	0.0334975	0.0773172	0.0205419	0.0229766	0.0211540	0.0035851
	0.4	0.0209260	0.0398772	0.0918409	0.0243201	0.0270822	0.0247950	0.0041738
	0.5	0.0221658	0.0422150	0.0971107	0.0256699	0.0285169	0.0260293	0.0043653
	0.6	0.0209260	0.0398772	0.0918409	0.0243201	0.0270822	0.0247950	0.0041738
	0.7	0.0175590	0.0334975	0.0773172	0.0205419	0.0229766	0.0211540	0.0035851
	0.8	0.0125944	0.0240515	0.0556298	0.0148260	0.0166523	0.0154113	0.0026282
	0.9	0.0065639	0.0125439	0.0290543	0.0077597	0.0087401	0.0081171	0.0013900
	1	0	0	0	0	0	0	0

the corresponding $\bar{\omega}_1$ and $\bar{\omega}_2$ are close to the natural frequency of the pure sdof system (only spring and mass) and the first natural frequency of the bare beam, respectively. This case suggests that the first mode of the coupled system is caused by the sdof system, and the other modes are similar with the ones of the bare beam.

In Fig. 8, it is noticed that a mode shape is absent and substituted by the next one when $a/L = 0.5$. Although the coupled system is a symmetric structure in the case, the anti-symmetric modes should still exist. Therefore the presented method would be further researched to explain the phenomenon.

7. Conclusions

In this paper, a theoretical model is developed to predict the vibration response of the coupled beam-sdof system. The transverse vibration expressions of this kind of system are formulated by using the recurrence equation method. Based on the recurrence equation, the natural frequencies and the corresponding mode shapes of the coupled system are also addressed. In addition, numerical examples of the undamped system are

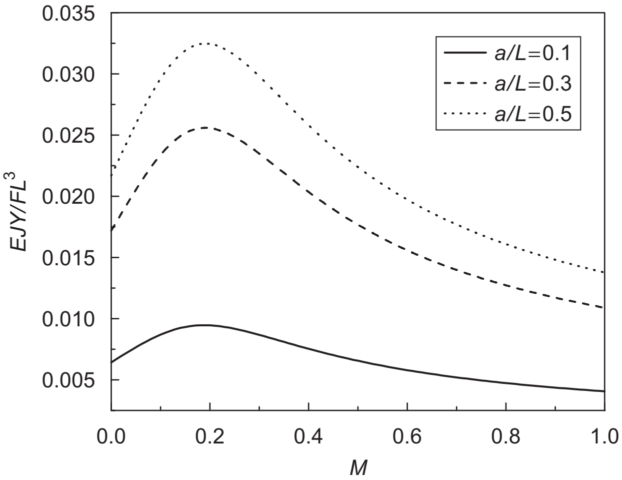

Fig. 2. $EJY(x)/FL^3$ versus M at $x = 0.5$, where $K = 1$, $\omega/\omega_1 = 0.2$ and $\xi = 0.5$.

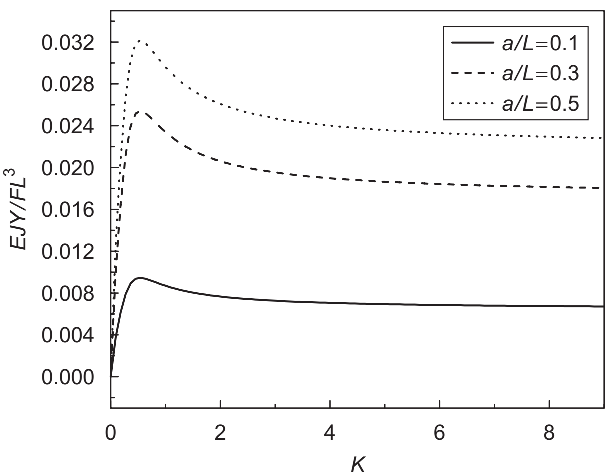

Fig. 3. $EJY(x)/FL^3$ versus K at $x = 0.5$, where $M = 0.1$, $\omega/\omega_1 = 0.2$ and $\xi = 0.5$.

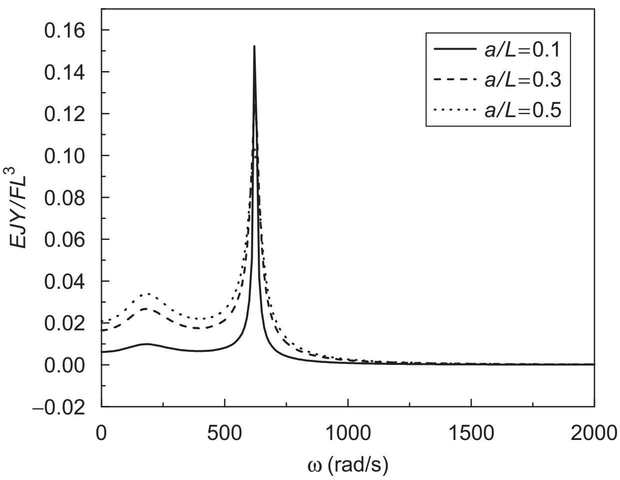

Fig. 4. $EJY(x)/FL^3$ versus ω at $x = 0.5$, where $M = 0.1$, $K = 1$ and $\xi = 0.5$.

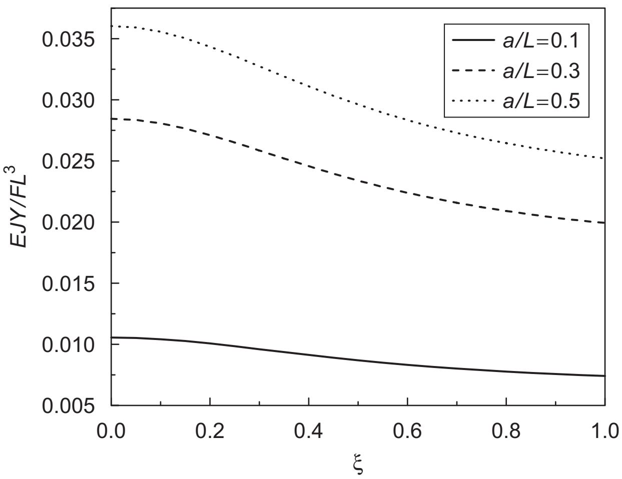

Fig. 5. $EJY(x)/FL^3$ versus ξ at $x = 0.5$, where $M = 0.1$, $\omega/\omega_1 = 0.2$ and $K = 1$.

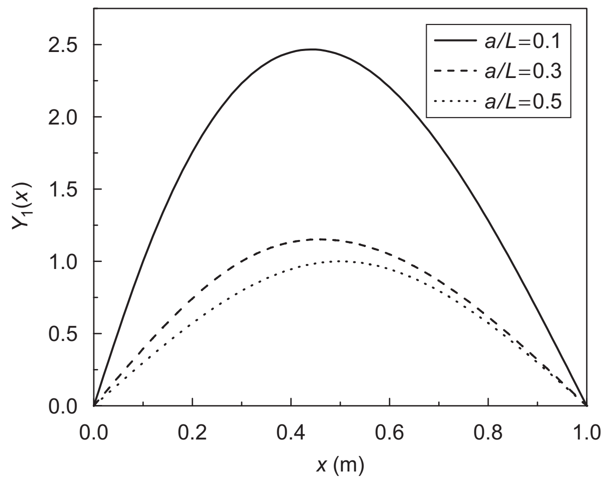

Fig. 6. The 1st mode shapes where $M = 0.1$ and $K = 2(\bar{\omega}_1 = 281.94$, 277.88 and 275.81 rad/s when $a/L = 0.1$, 0.3 and 0.5, respectively).

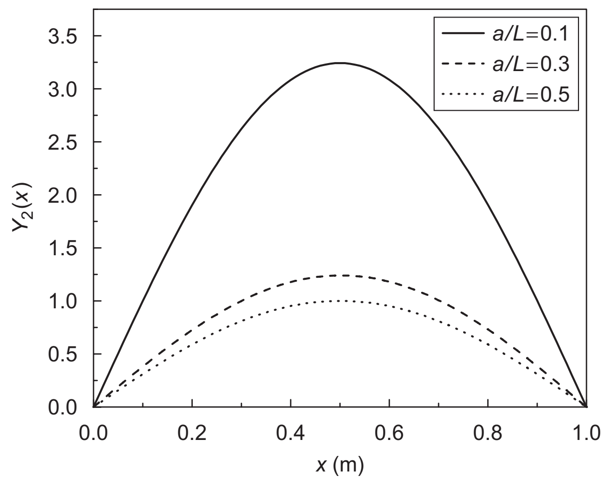

Fig. 7. The 2nd mode shapes where $M = 0.1$ and $K = 2(\bar{\omega}_2 = 625.74$, 634.56 and 639.93 rad/s when $a/L = 0.1$, 0.3 and 0.5, respectively).

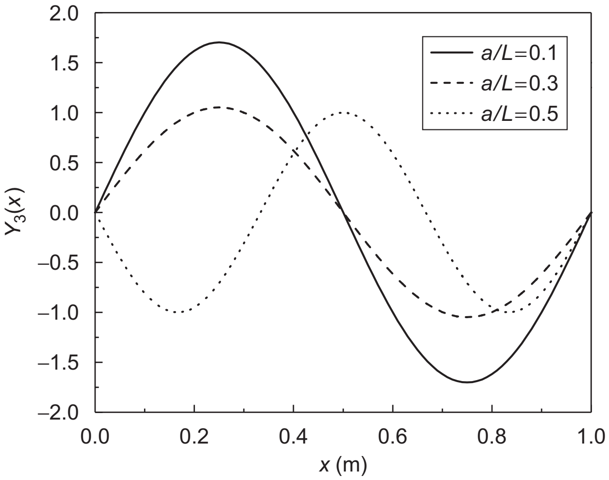

Fig. 8. The 3rd mode shapes where $M = 0.1$ and $K = 2(\bar{\omega}_3 = 2497.95$, 2499.77 and 5619.31 rad/s when $a/L = 0.1$, 0.3 and 0.5, respectively).

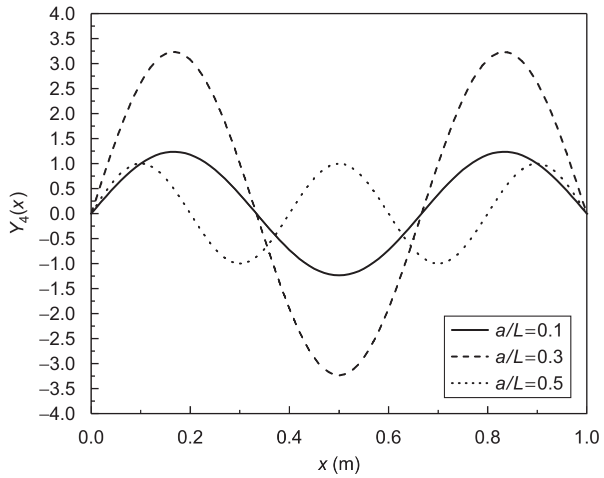

Fig. 9. The 4th mode shapes where $M = 0.1$ and $K = 2(\bar{\omega}_4 = 5618.81$, 5618.01 and 15605.73 rad/s when $a/L = 0.1$, 0.3 and 0.5, respectively).

given and compared with the published results. The agreement is excellent which shows the study work in this paper is correct. Subsequently, the parametric influence analyses for the coupled system are presented. To show the utility of the present approach, the lowest four natural frequencies and mode shapes of the coupled system are calculated. The theory and model in this paper is an attempt in order to introduce the recurrence equation method into the field of vibration analysis and achieve the preliminary study for the prediction of sound radiation from the coupled system. The absence of anti-symmetric modes for the symmetric system as well as the vibration analysis of the beam-sdof system with other boundary conditions will be investigated in the future work.

Acknowledgements

The work described in this paper was supported by Program for New Century Excellent Talents in University of China (Approval No. NCET–06–0842).

References

[1] J.S. Wu, T.L. Lin, Free vibration analysis of a uniform cantilever beam with point masses by an analytical-and-numerical-combined method, *Journal of Sound and Vibration* 136 (1990) 201–213.

[2] J.S. Wu, D.W. Chen, Dynamic analysis of a uniform cantilever beam carrying a number of elastically mounted point masses with dampers, *Journal of Sound and Vibration* 229 (2000) 549–578.

[3] M. Gürgöze, On the alternative formulations of the frequency equations of a Bernoulli–Euler beam to which several spring–mass systems are attached in span, *Journal of Sound and Vibration* 217 (1998) 585–595.

[4] S. Inceoğlu, M. Gürgöze, Longitudinal vibrations of rods coupled by several spring–mass systems, *Journal of Sound and Vibration* 234 (2000) 895–905.

[5] S. Inceoğlu, M. Gürgöze, Bending vibration of beams coupled by several double spring–mass systems, *Journal of Sound and Vibration* 243 (2001) 370–379.

[6] D.V. Bambill, C.A. Rossit, Forced vibrations of a beam elastically restrained against rotation and carrying a spring–mass system, *Ocean Engineering* 29 (2002) 605–626.

[7] M.C.Junger, D.Feit, *Sound, Structure, and their Interaction*, second ed., The MIT Press, Cambridge, MA, 1986, pp. 206–209.